CS 61A SU26

Lecture 03: Higher-Order Functions (HOFs)

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Join pollev.com/rebeccadang831 (or scan QR code) on your phone or laptop

We'll begin at Berkeley time (10 minutes after), as per tradition!

Potpourri

5 min

- Announcements
- Quizzes
- Review: Control

Announcements

- Lab 01, HW 01, and Hog (first project) are released!
- Projects in this course:
  - Checkpoint due date
  - Early due date (extra credit!)

Quizzes

- Highly recommend: Do orientation quiz ASAP if you haven't already
  - If you miss it, CBTF will release remote, async quiz next week
- Reserve Quiz 1 slot ASAP
  - DSP accommodations: **Please submit your letters AND login to PrairieTest (PT) ASAP** so we can process your accommodations **before** you reserve your slot
  - Accommodations do **not apply retroactively** if you already reserved
  - **If you already reserved, check that your accommodations were factored in on PT.** It should show up on the home page where it shows your reservation time.
- Quiz 1 logistics and practice quiz to be released (ideally) tomorrow

When poll is active respond at **PollEv.com/rebeccadang831**

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Select ALL of the ways to fill in blanks A and B that would result in EQUIVALENT behavior when Variation 1 and 2 are executed independently (regardless of the value of x).

Fibonacci

- The Fibonacci sequence is defined as:
  - 0th Fibonacci number is 0
  - 1st Fibonacci number is 1
  - Fibonacci number n is the sum of the previous 2 numbers
- Ex: 0, 1, 1, 2, 3, 5, 8, 13, ...

Wikipedia: "**The Fibonacci numbers were first described in Indian mathematics as early as 200 BC in work by Pingala** on enumerating possible patterns of Sanskrit poetry formed from syllables of two lengths. They are **named after** the **Italian mathematician Leonardo of Pisa, also known as Fibonacci**, who **introduced the sequence to Western European mathematics** in his 1202 book *Liber Abaci*."

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Select ALL implementations of Fibonacci that are correct.

Higher-Order Functions

50 min

- Functions as data
- Lambda expressions
- HOFs
- Functions as inputs
- Functions as outputs
- Lambdas: WWPD
- Environments

Functions as data

- Unlike some other programming languages, in Python, functions can be treated as both **procedures** and **data**
  - Functions are **"first-class" objects** (more on objects later!)
- Ex: In lecture 1, we "stored" `min` and `max` in variables, `f` and `g`
- Demo: `<function object>`

Lambda expressions

- So far, we learned to use `def` to define our own functions
- Another way: **Lambda expressions**
  - `lambda <parameters>: <return expression>`
- Lambda functions are often called **anonymous functions** because they have no **intrinsic name** (unlike functions created with `def`)
- Demo
  - Lambda with 0 or more parameters
  - Storing and calling a lambda function

HOFs

- A **higher-order function (HOF)** is a function that takes in another function as input, returns a function as output, or both
- Motivation:
  - Generalization
  - Avoiding repetition
    - DRY: **D**on't **R**epeat **Y**ourself
  - "Factory" functions, e.g. `make_discount` (later slide)

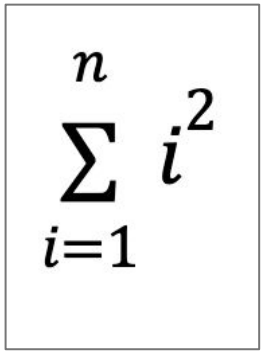
Functions as inputs (1 of 3)

```
def sum_of_squares(n):
    i = 1
    total = 0
    while i <= n:
        total += i ** 2
        i += 1
    return total
```

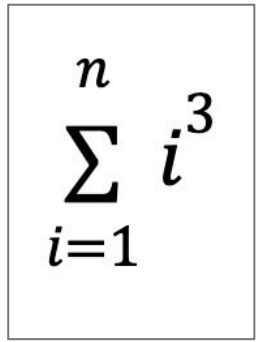
```
def sum_of_cubes(n):
    i = 1
    total = 0
    while i <= n:
        total += i ** 3
        i += 1
    return total
```

Functions as inputs (2 of 3)

```
def sum_of_squares(n):
    i = 1
    total = 0
    while i <= n:
        total += i ** 2
        i += 1
    return total
```




```
def sum_of_cubes(n):
    i = 1
    total = 0
    while i <= n:
        total += i ** 3
        i += 1
    return total
```



Functions as inputs (3 of 3)

```
def summation(n, term):
    i = 1
    total = 0
    while i <= n:
        total += term(i)
        i += 1
    return total
```

Functions as outputs (1 of 2)

```
def half_off(price):
    return price * 0.5

def ten_percent_off(price):
    return price * 0.9

def twenty_percent_off(price):
    return price * 0.2
```

```
def compute_new_price(discount, price):
    return discount(price)
```

Functions as outputs (2 of 2)

```
def make_discount(percent_off):
    def discount(price):
        return price * (1 - percent_off)
    return discount
```

```
def compute_new_price(discount, price):
    return discount(price)
```

```
>>> half_off = make_discount(0.5)
>>> compute_new_price(half_off, 20)
10.0
```

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Lambdas: What would Python display?

Lambdas WWPD: What happened? (1 of 5)

```
(lambda x, y: lambda z: x * y + z)(1, 2)(3)
```

Outer function called directly

Operands of outer function (x and y)

Lambdas WWPD: What happened? (2 of 5)

```
(lambda x, y: lambda z: x * y + z)(1, 2)(3)
```

Return expression of outer function

Lambdas WWPD: What happened? (3 of 5)

```
(lambda x, y: lambda z: x * y + z)(1, 2)(3)
```

Since the inner function was returned, this entire thing evaluates to that inner function

Lambdas WWPD: What happened? (4 of 5)

```
(lambda x, y: lambda z: x * y + z)(1, 2)(3)
```

Return expression of inner function (has access to x and y from its parent frame)

Operand of inner function (z)

Lambdas WWPD: What happened? (5 of 5)

```
(lambda x, y: lambda z: x * y + z)(1, 2)(3)
```

```
def outer(x, y):
    def inner(z):
        return x * y + z
    return inner
outer(1, 2)(3)
```

Tip: Rewrite `lambda` into `def` if you're confused on what it's doing! Or do the opposite if we ask you to write a nested `lambda`.

Environments

- In a previous slide, we talked about "parent frame"
  - But what's a parent? And what's a frame?
  - How do we figure out what data we have access to in a program? (e.g. **variable scope**)
- An **environment** is a sequence of **frames**
  - You might also hear programmers call it a "**stack**" of frames, hence the term **"stack trace"** from Lecture 1: Exceptions, or **"stack overflow"** (we'll talk about that next week!)
- A **frame** is a set of bindings between names and values
- All Python programs begin in the special **global frame**

Name Lookup Rules

- A function's **parent frame** is the **frame in which the function is <u>defined</u>** (not the frame in which it's called)
- Each time we call a function, we open a new **local frame**
- When we try to retrieve the value of a name, we first look in the current frame
  - If it's not there, we recursively check if it exists in the **parent frame**
  - If we go all the way up to the global frame and it's still not there, throw a `NameError`
- ⚠ **Caution:** You can't update a variable outside your local frame (results in `UnboundLocalError`)

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Environments: What would Python display?

5 min break

HOFs Practice

20 min

- Combination
- Multiply Triple

Practice: Combination

- Implement a higher-order function `combination(n, initial, combiner, term)` See the doctests for examples.
- Download starter code `03.py` from the course website under Lecture 3
- Run doctests with `python3 -m doctest 03.py`
- 5 min: Try implementing it yourself
- 5 min: Discuss with someone next to you and compare your implementations
  - What worked?
  - What didn't?
  - Why?

Practice: Multiply Triple

- **Using only functions with a single argument**, implement the function `multiply_triple` so that you can use it to return the product of 3 numbers x, y, and z.
- Download starter code `03.py` from the course website under Lecture 3
- Run doctests with `python3 -m doctest 03.py`
- 5 min: Try implementing it yourself
- 5 min: Discuss with someone next to you and compare your implementations
  - What worked?
  - What didn't?
  - Why?